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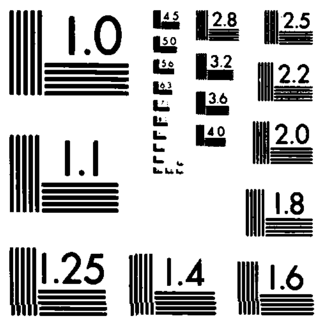

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ABSTRACT

Single component plasmas are found to exhibit a reversal in the roles of cyclotron and upper hybrid resonances familiar in neutral plasmas. There is a global resonance at the cyclotron frequency $\omega = \Omega \equiv qB/mc$ and a local field cancellation at layers where $\omega = \Omega_1 = (\Omega^2 - \omega_p^2)^{1/2}$, ω_p being the local plasma frequency. Thermal effects limit the amplitude of the global resonance and produce energy absorption at the $\omega = \Omega_1$ layers.

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Single component plasmas confined by external magnetic fields strong enough to overcome the Coulomb self-repulsion have been demonstrated[1,2] to exhibit remarkably long lifetimes and to be rather quiescent compared to other laboratory plasmas. Such properties make these systems ideally suited for the study of effects in which the presence of a self-consistent electrostatic field plays an important role. A proper understanding of wave and transport phenomena including the effects of such self-electric fields is essential for developing a better description of conventional "neutral" plasmas, which in reality develop strong ambipolar electric fields, due to the different transport properties of electrons and ions. The present theoretical study examines some unusual properties of an extreme nonneutral system, namely a single component plasma, with special emphasis on the plasma response near the cyclotron frequency. The effects analyzed may also be of interest in understanding cyclotron resonance absorption and/or emission in plasmas confined in magnetic mirror configurations, where electrostatic fields are known to be present.

In a cold magnetized neutral plasma, cross-field plasma behavior near the electron cyclotron frequency Ω is governed by the dielectric tensor element $\varepsilon_{xx} = 1 - \omega_p^2/(\omega^2-\Omega^2) = (\omega^2-\omega_{uh}^2)/(\omega^2-\Omega^2)$ where ω_p is the electron plasma frequency and $\omega_{uh} = (\Omega^2 + \omega_p^2)^{1/2}$ the upper hybrid frequency. Two features of ε_{xx} that have important consequences for the plasma response are $\varepsilon_{xx} \to \infty$ as $\omega \to \Omega$ and $\varepsilon_{xx} \to 0$ as $\omega \to \omega_{uh}$. For a neutral plasma slab in which the gradient in the density $n_o(x)$ is perpendicular to a uniform magnetic field $B\hat{z}$, the application of a capacitor plate field $E_c\hat{x}$ at a frequency ω results in a collective field $E_1 = E_c/\varepsilon_{xx}$ which vanishes at <u>all</u> x when $\omega = \Omega$ and has a local resonance ($E_1 \to \infty$) at the upper hybrid layers where $\omega = \omega_{uh}(x)$.

For a single component plasma we find a strikingly different behavior.

The dielectric tensor element ε_{xx} for a cold single component plasma can be obtained as the lowest order term in a kinetic treatment which involves integrating the linearized Vlasov equation along the orbit of a particle moving under the influence of the self-electric field $E_o(x)\hat{x}$ and the uniform magnetic field $B\hat{z}$. Assuming the cyclotron radius $r_c << L \equiv |(d/dx)\ln n_o|^{-1}$ and retaining only terms linear in r_c/L in the electric field

$$E_o(x') = E_o(x) + dE_o/dx\ (x'-x) = E_o(x) + 4\pi q n_o(x)(x'-x) \qquad (1)$$

governing the orbit $\underset{\sim}{x}'(\tau)$ of a particle starting from $\underset{\sim}{x}$, we find that the linear term in the expansion tends to weaken the magnetic restoring force and the particle gyrates not at angular frequency Ω, but at $\Omega_1(x) \equiv [\Omega^2-\omega_p^2(x)]^{1/2}$. Hence, for a single component plasma, $\varepsilon_{xx} = 1 - \omega_p^2/(\omega^2-\Omega_1^2) = (\omega^2-\Omega^2)/(\omega^2-\Omega_1^2)$. Comparison with the neutral plasma result shows that the familiar roles of single particle resonance and collective resonance are in a sense interchanged; we now have $\varepsilon_{xx} \to 0$ as $\omega \to \Omega$ and $\varepsilon_{xx} \to \infty$ at layers where $\omega = \Omega_1(x) = (\Omega^2-\omega_p^2)^{1/2}$. The collective field E_1 exhibits a global resonance (i.e., $E_1 \to \infty$ for all x) when $\omega = \Omega$ and a local null at layers where $\omega = \Omega_1(x)$.

As expected, finite temperature effects modify both the global resonance and the local null of E_1. We now discuss two such effects. The first one, characteristic of nonneutral plasmas, arises from the modification of the single particle orbits when higher order terms in r_c/L are included in the expansion (1) for the equilibrium electric field $E_o(x')$. The second one is associated with the existence of collective modes and can be regarded as a

correction to ϵ_{xx} in powers of kr_c where $k \equiv |d/dx \ln E_1|$, E_1 being the linear wave field. To evaluate both of these effects, we need to choose a suitable equilibrium distribution function $f_0(\underset{\sim}{x},\underset{\sim}{v})$ constructed, of course, from the constants of the motion. We find that the choice $f_0 = C \exp[-(v^2+2q\phi_0(x)/m)/2\bar{v}^2 - (x + v_y/\Omega)^2/2\ell^2]$, where $\bar{v}$ and ℓ are given parameters and C is a normalization constant, provides a convenient bell-shaped density profile, peaked at x=0. The function f_0 is a bi-Maxwellian in a local frame drifting with velocity $\underset{\sim}{v}_d(x) = -x\Omega r_c^2/(r_c^2+\ell^2)\hat{y}$ which is the sum of the $\underset{\sim}{E}\times\underset{\sim}{B}$ and the diamagnetic drift velocities; the mean squared velocity components are $\langle v_x^2\rangle = \langle v_z^2\rangle = \bar{v}^2$, $\langle(v_y - v_d)^2\rangle = \bar{v}^2\ell^2/(r_c^2+\ell^2)$. Note that the equilibrium electric potential ϕ_0 appearing in f_0 must be self-consistently determined from Poisson's equation which can be written in the dimensionless form $d^2\psi/d\xi^2 = e^{\psi} - A$, where $\psi = -q[\phi_0(x)-\phi_0(0)]/m\bar{v}^2 - A\xi^2/2$, $\xi = x/\lambda_D$, $\lambda_D = \bar{v}/\omega_p(0)$, $A = \lambda_D^2/(r_c^2 + \ell^2)$. The parameter A, which determines the density profile $n_0(\xi)/n_0(0) = e^{\psi}$, must be greater than unity for a bounded plasma [$n_0 \to 0$ as $\xi \to \infty$]. The self-consistent potentials and the corresponding density profiles for A=1.1, 2.0 are displayed in Fig. 1. For A in this range, the density scale length L is of the order of the Debye length λ_D and so the assumption $r_c \ll L$ implies $\omega_p(0) \ll \Omega$; this, in turn, implies that the single particle resonance frequency $\Omega_1(x)$ is very close to the cyclotron frequency Ω for all x.

The first of the finite temperature effects arises from the inclusion of the $(r_c/L)^2$ correction term $(1/2)(d^2E_0/dx^2)(x'-x)^2$ in Eq.(1) in calculating the single particle orbits. The gyration frequency of a particle with initial position $\underset{\sim}{x}$ and velocity $\underset{\sim}{v}$ is Ω_1, evaluated at the guiding center position $x_g \simeq x + (qE_0/m+\Omega v_y)/\Omega^2$. Thus the gyration frequency is modified from

$\Omega_1(x)=[\Omega^2 + \frac{q}{m}E_o'(x)]^{1/2}$ to

$$\Omega_2(x,v_y)=[\Omega^2+ \frac{q}{m}E_o'(x_g)]^{1/2}=\Omega_1(x)+ \frac{q}{m}E_o''(x)[\frac{qE_o(x)/m+\Omega v_y}{2\Omega^3}] \tag{2}$$

Solving the linearized Vlasov equation by integration over particle trajectories, we find

$$\varepsilon_{xx} = 1 + \frac{1}{2}\frac{\omega_p^2}{\omega\Delta\omega}\left[Z\left(\frac{\omega + \Omega_1}{\Delta\omega}\right) + Z\left(\frac{\omega - \Omega_1}{\Delta\omega}\right)\right] \tag{3}$$

where $\Delta\omega = r_c \ell |d\omega_p^2/dx| [2\Omega^2(r_c^2 + \ell^2)]^{-1/2}$ and Z is the plasma dispersion function.[3] When $|\omega - \Omega_1|>>\Delta\omega$, ε_{xx} reduces to the cold plasma result, but at layers where $\omega=\Omega_1(x)$ it remains finite and E_1 no longer vanishes there. Furthermore, ε_{xx} is complex, corresponding to energy transfer from the field to the particles. This energy transfer, which arises from the velocity dependence of Ω_2, is analogous to the familiar cyclotron damping associated with the Doppler shift in cyclotron resonance from Ω to $\Omega + k_z v_z$ when the parallel wave number $k_z \neq 0$. [In fact, allowing finite k_z in the present study would result in additional cyclotron damping, which would be negligible for $k_z\bar{v} << \Delta\omega$ or $k_z^{-1}>>L(\Omega/\omega_p)^2$]. Fig. 2 displays the real and imaginary parts of the electric field $E_1 = E_c/\varepsilon_{xx}$ [ε_{xx} given by Eq.(3)] driven by an external antenna at $\omega/\Omega=.9975$ for a plasma with $\omega_p(0)/\Omega = .1$, A = 1.1. The spatial width of the resonance layers where Im E_1 is significant is of the order of r_c, so the power absorbed per unit area of each resonance layer is $P/a \sim r_c(\omega/8\pi)\ |E_1|^2 \mathrm{Im}\ \varepsilon_{xx}$ corresponding to a local absorption rate $P/[ar_c E_c^2/8\pi] \sim (r_c/L)\Omega$.

The inclusion of $(r_c/L)^2$ corrections to the particle orbits removes not only the local null of E_1 at layers where $\omega = \Omega_1(x)$ but also the global singularity of E_1 as $\omega \to \Omega$ (except where $dn_o/dx = 0$). The global singularity in E_1 at $\omega = \Omega$ is also removed (even where $dn_o/dx = 0$) by a different thermal effect, namely the inclusion of finite kr_c corrections to ε_{xx}. This converts ε_{xx} into a differential operator. Keeping the lowest order corrections [which involve $(kr_c)^2$], we obtain a second order differential equation for E_1,

$$- r_c^2 \frac{d}{dx} \left\{ \omega_p^2(x) \frac{d}{dx} \left[E_1 \frac{\Omega^2}{\Omega_1^2} \left(\frac{1}{\omega^2 - \Omega_1^2} - \frac{1}{\omega^2 - 4\Omega_1^2} \right) \right] \right\} + \left(1 - \frac{\omega_p^2}{\omega^2 - \Omega_1^2} \right) E_1 = E_c \qquad (4)$$

where, in order to emphasize the finite kr_c effects, we have suppressed the Z functions. Eq.(4) predicts the existence of collective thermal modes with a WKB dispersion relation $(kr_c)^2 = (\omega^2 - \Omega^2)(\omega^2 - 4\Omega_1^2)/3\Omega^2\omega_p^2$, propagating for $\omega < \Omega$, $\omega > 2\Omega_1$ with cutoffs at $\omega = \Omega$, $2\Omega_1$. These modes are analogous to Bernstein waves of neutral plasmas. The Bernstein dispersion relation for a single species plasma has been obtained before by Davidson[4] for the special case of a cylindrical rigid rotor and is identical to the neutral plasma dispersion relation except for the shift $\Omega \to \Omega - 2\omega_e$ (where ω_e is the rigid rotor angular velocity) caused by the Coriolis force. In our slab problem, however, the Coriolis force is absent and the shift $\Omega \to \Omega_1$ is caused by the gradient in the electric field (corresponding to a shear in the angular velocity for the cylindrical case).

The thermal mode electric field for the undriven case ($E_c = 0$) obtained by numerical integration of Eq. (4) is displayed in Fig. 3 for the choice $\omega/\Omega = .9975$, $\omega_p(0)/\Omega = .1$, $A = 1.1$. (Note that the condition $kr_c < 1$ for which Eq. (4) is valid, requires frequencies close to the cutoff values.) In a driven system ($E_c \neq 0$), the thermal mode pattern appears superposed on the cold plasma response. In a slab of finite width ($-x_b < x < x_b$), the thermal modes can be

strongly excited at those frequencies ω_n for which the undriven solution has nodes at $x = \pm x_b$. These are the analogues of the Tonks-Dattner resonances[5] in neutral plasmas.

Note also that the existence of the thermal modes removes the cold plasma global resonance at the frequency $\omega = \Omega$. Integrating Eq. (4) twice for $\omega = \Omega$, we obtain

$$E_1(x) = E_c\ \omega_p^2 \left[\frac{1}{\omega_p^2(x_b)} + \frac{1}{r_c^2} \int_x^{x_b} \frac{x'dx'}{\omega_p^2(x')}\right] \tag{5}$$

so that $E_1(0) \sim (x_b/r_c)^2 \left[\omega_p^2(0)/\omega_p^2(x_b)\right] E_c$ is large but finite.

In summary, the behavior of a single component plasma for frequencies in the neighborhood of the cyclotron frequency differs significantly from the neutral case. New phenomena are available for theoretical and experimental investigation that can provide deeper insight into the nature of cyclotron resonance heating and hybrid resonances in magnetically confined plasmas.

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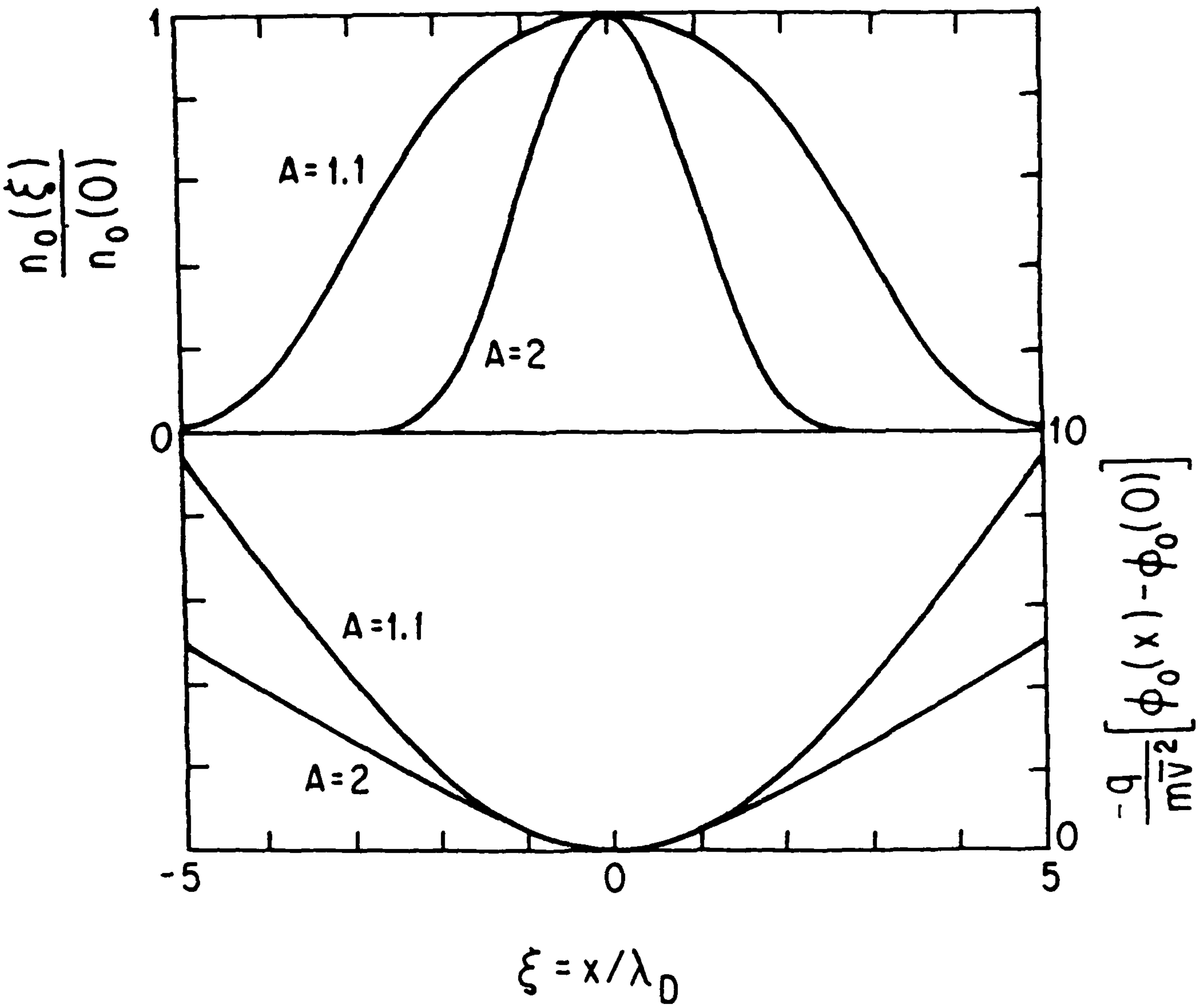
FIGURE CAPTIONS

Fig. 1. Self-consistent density profiles (top) and corresponding potentials (bottom) in a single component plasma for A = 1.1, 2.0.

Fig. 2. Real and imaginary parts of the electric field $E_1 = E_c/\varepsilon_{xx}$ [ε_{xx} given by Eq. (3)] driven by an external field E_c for A = 1.1, $\omega_p(0)/\Omega = 0.1$, $\omega/\Omega = 0.9975$.

Fig. 3. Thermal mode electric field for the undriven case ($E_c = 0$) for A = 1.1, $\omega_p(0)/\Omega = 0.1$, $\omega/\Omega = 0.9975$.



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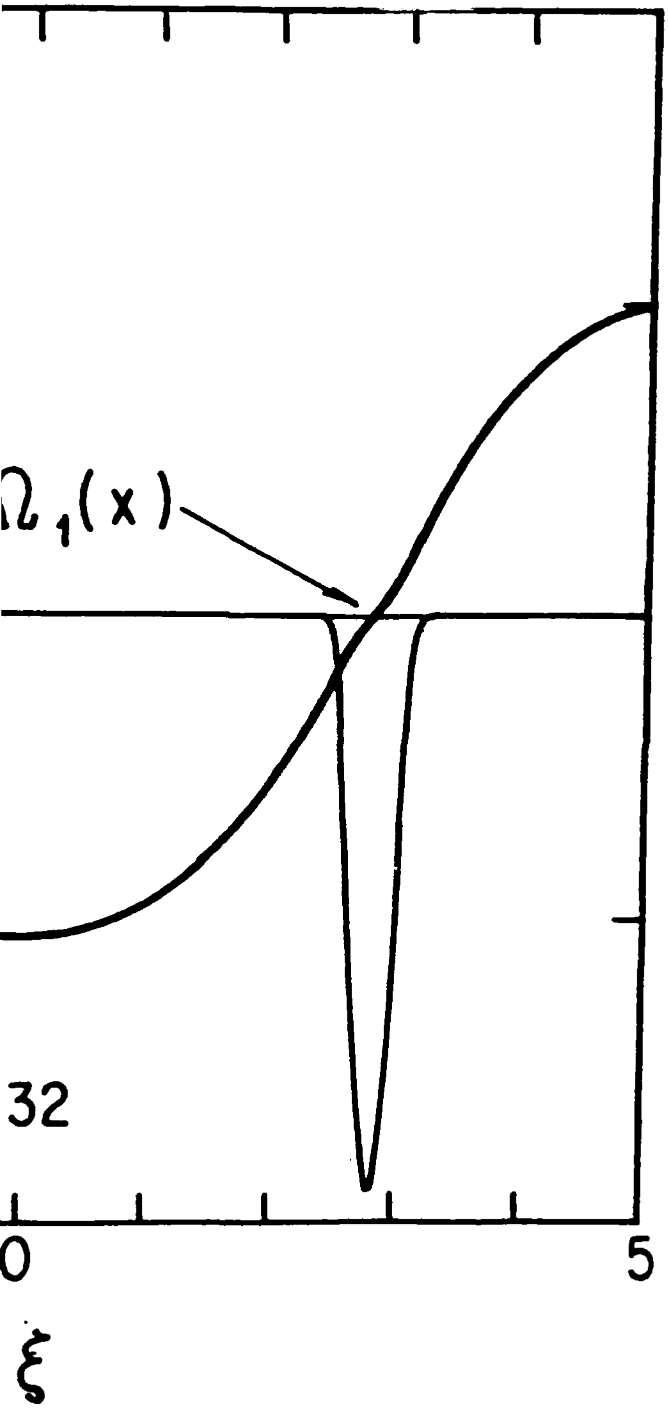

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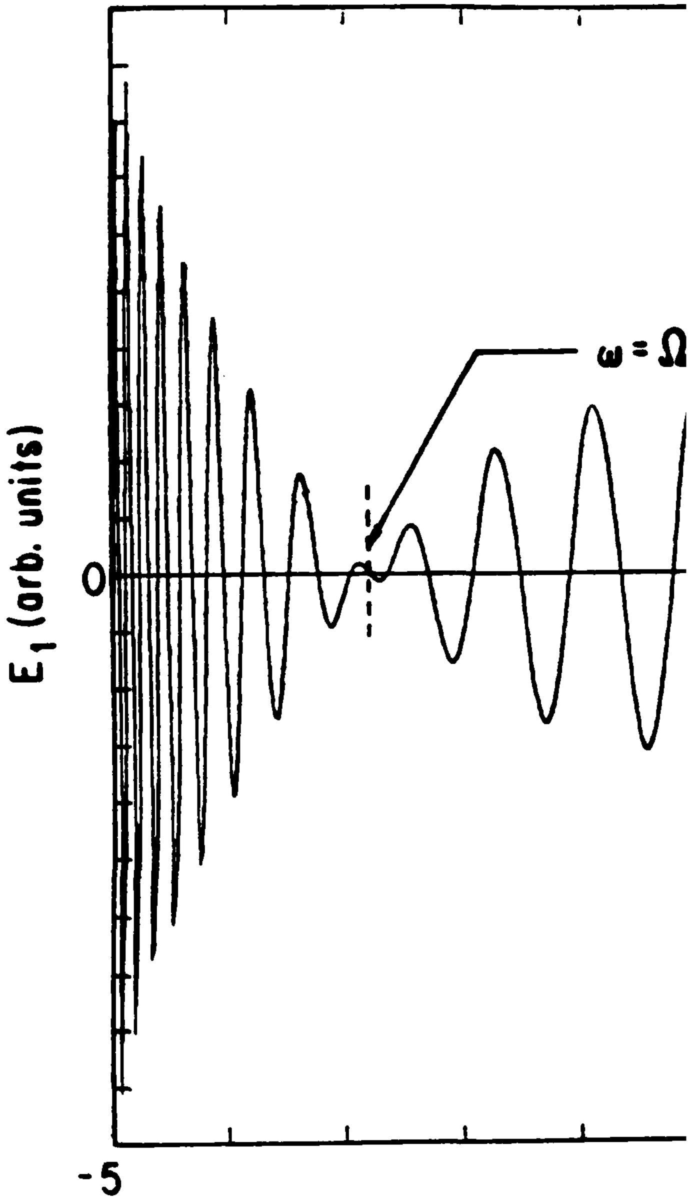

Fig. 3. Thermal mode electric field

$A = 1.1$, $\omega_p(0)/\Omega = 0.1$, ω/Ω

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